Analysis and Control Software for Distributed Cooperative Systems

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Analysis and Control Software for Distributed Cooperative Systems

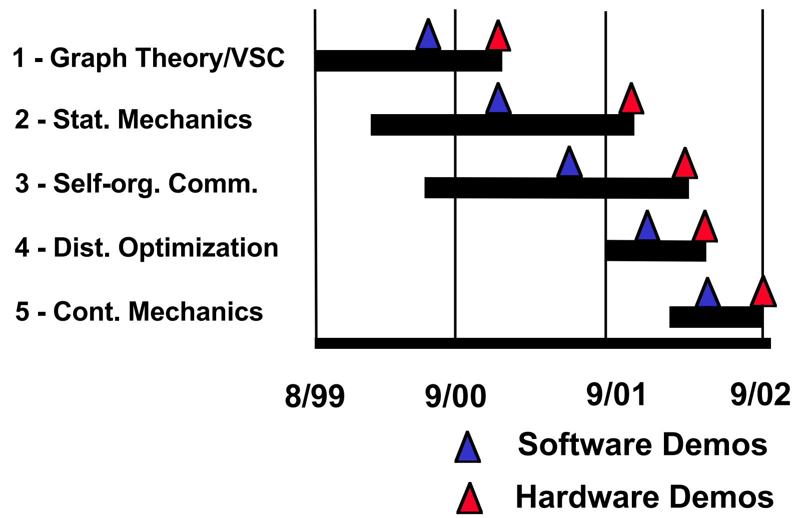
New Ideas

- Large-scale analysis and control of distributive cooperative systems using graph theory and variable structure control.
- Self-organizing communication networks.
- Distributed optimization formulation of tasking and control problems.
- Very large-scale system analysis using statistical and continuum mechanics.

Impact

- Force Multiplier - Enables a single operator to control 100s to 1000s of distributed systems.
- Provably convergent control algorithms ensure safe operations.
- Distributed controls provides fault tolerance.
- Low communication bandwidth for covert operations.

Schedule



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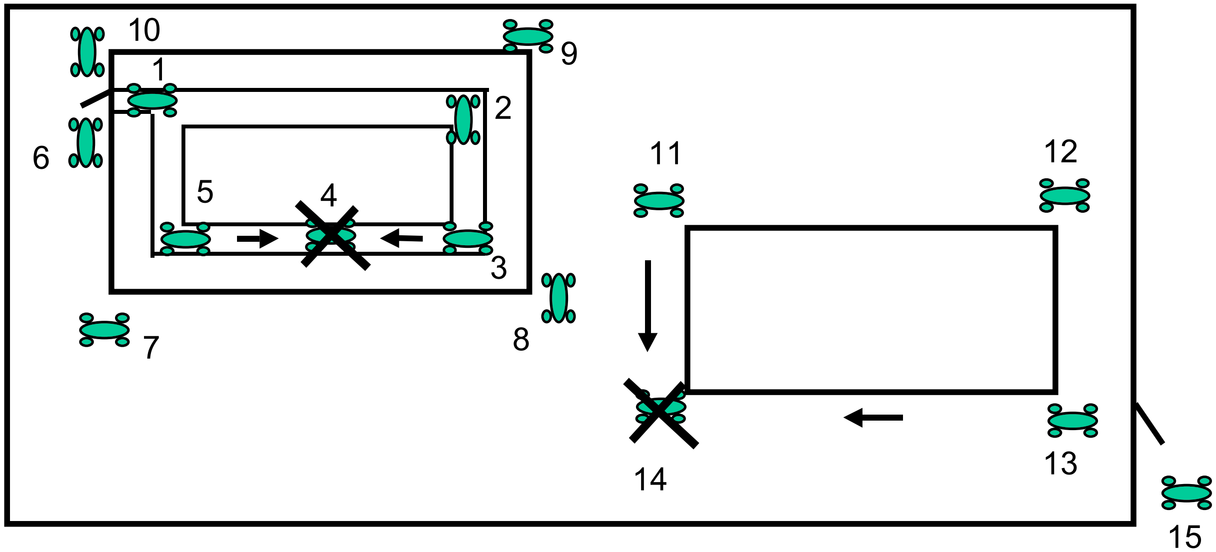
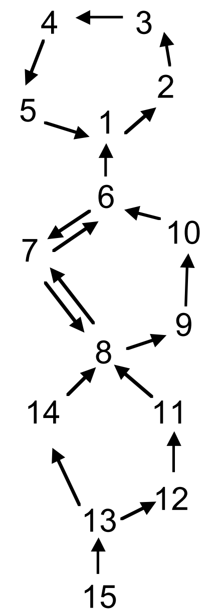
Presentation Outline

- Stability analysis.
- How analysis applies to real problems.
- Implementation in progress.
- Proposed future work.

Communication/Navigation Network





- Use state space models and graph theory to
  - Determine strongly connected subsystems.
    - Input/output reachable.
    - Structurally observable/controllable.
    - Connectively stable.
  - Evaluate reconfigurable mobile communication networks.

Stability Analysis of Large Scale Systems

- Computing controllability and observability is numerically difficult for large scale systems. Instead, we compute
  - Input Reachability => Structural Observability
  - Output Reachability => Structural Controllability
  - Vector Liapunov => Connective Stability

Large Scale Systems

- State Space Model of N interconnected subsystems:

$$\mathbf{S}: \quad \dot{x}_i = f_i(t, x_i, u_i) + \tilde{f}_i(t, x, u), \qquad i \in \{1, \ldots, N\}$$
$$y_i = h_i(t, x_i) + \tilde{h}_i(t, x)$$

where

$$\tilde{f}_i(t, x, u) = \tilde{f}_i\left(t, \bar{a}_{i1}x_1, \bar{a}_{i2}x_2, \ldots, \bar{a}_{iN}x_N, \bar{b}_{i1}u_1, \bar{b}_{i2}u_2, \ldots, \bar{b}_{iN}u_N\right)$$
$$\tilde{h}_i(t, x) = \tilde{h}_i\left(t, \bar{c}_{i1}x_1, \bar{c}_{i2}x_2, \ldots, \bar{c}_{iN}x_N\right)$$

and $\bar{a}_{ij}, \bar{b}_{ij}, \bar{c}_{ij}$ are 1 or 0 (coupling or no coupling).

- Control feedback is added to system such that

$$u_i = k_i(t, y_i) + \tilde{k}_i(t, y), \qquad i \in \{1, \ldots, N\}$$
$$\tilde{k}_i(t, y) = \tilde{k}_i\left(t, \bar{k}_{i1}y_1, \bar{k}_{i2}y_2, \ldots, \bar{k}_{iN}y_N\right)$$

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Large Scale Systems

- Interconnection Matrix is

$$E = \begin{bmatrix} \overline{A} & \overline{B} & 0 \\ 0 & 0 & \overline{K} \\ \overline{C} & 0 & 0 \end{bmatrix} \quad \text{where} \quad \overline{A} = (\overline{a}_{ij}) \quad \overline{B} = (\overline{b}_{ij}) \quad \overline{C} = (\overline{c}_{ij}) \quad \overline{K} = (\overline{k}_{ij})$$

- Reachability Matrix is $R = E \vee E^2 \vee \ldots \vee E^s = \begin{bmatrix} F & G & 0 \\ 0 & 0 & 0 \\ H & \theta & 0 \end{bmatrix}$ if $\overline{K} = 0$

– Input reachable iff G has nonzero rows.
– Output reachable iff H has nonzero rows.
– Structurally controllable if input reachable and no dilations (independent control all state variables).
– Structurally observable if output reachable and no dilations.

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Large Scale System Stability

- Closed loop dynamics

$$\mathbf{S}: \quad \dot{x}_i = g_i(t, x_i) + \tilde{g}_i(t, x), \qquad i \in \{1, \ldots, N\}$$

$$\tilde{g}_i(t, x) = \tilde{g}_i(t, \bar{e}_{i1}x_1, \bar{e}_{i2}x_2, \ldots, \bar{e}_{iN}x_N)$$

- Connectively stable if $W = (w_{ij})$ where $w_{ij} = \begin{cases} 1 - \bar{e}_{ii}\kappa_i\xi_{ii}, & i = j \\ -\bar{e}_{ij}\kappa_i\xi_{ij}, & i \neq j \end{cases}$ is an M-matrix (all leading principle minors must be positive). Variables $\kappa_i > 0$, $\xi_{ij} \geq 0$ must satisfy

$$|v_i(t, x') - v_i(t, x_i'')| \leq \kappa_i \|x_i' - x_i''\|, \quad \forall t \in T, \quad \forall x_i', x_i'' \in \Re^{n_i}$$

$$\|\tilde{g}_i(t, x)\| \leq \sum_{j=1}^{N} \bar{e}_{ij}\xi_{ij}\phi_j(\|x_j\|) \quad \forall (t, x) \in T \times \Re^n$$

$$\dot{v}_i(t, x_i) \leq -\phi_j(\|x_j\|) \quad \forall (t, x_i) \in T \times \Re^{n_i}$$

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Large Scale System Stability

- Closed loop dynamics for linear systems

$$\mathbf{S}: \quad \dot{x}_i = A_i x_i + \sum_{j=1}^{N} e_{ij} A_{ij} x_j, \qquad i \in \{1, \ldots, N\}$$

$$0 \le e_{ij} \le 1$$

- Connectively stable if $W = (w_{ij})$ is an M-matrix (all leading principle minors must be positive) where

$$w_{ij} = \begin{cases} \dfrac{\lambda_m(G_i)}{2\lambda_M(H_i)} - \bar{e}_{ii}\lambda_M^{1/2}\left(A_{ii}^T A_{ii}\right) & i = j \\ -\bar{e}_{ij}\lambda_M^{1/2}\left(A_{ij}^T A_{ij}\right) & i \ne j \end{cases}$$

$$A_i^T H_i + H_i A_i = -G_i$$

Note: $0 \le e_{ij} \le 1$ implies stable even if communication is down or degraded.

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Analysis of Example Problems

- Perimeter Surveillance
- Self-Healing Minefield
- Distributed Communication Navigation Network

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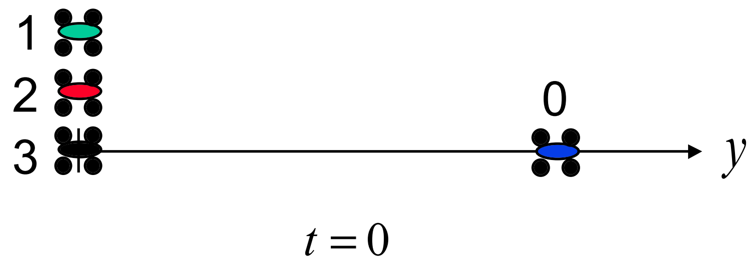
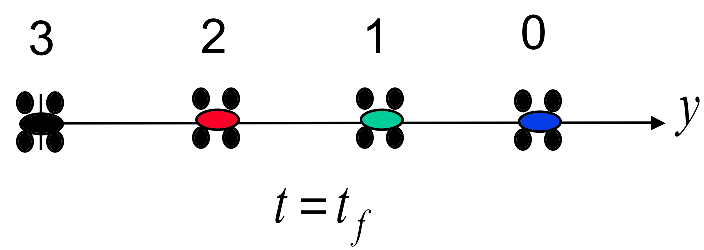
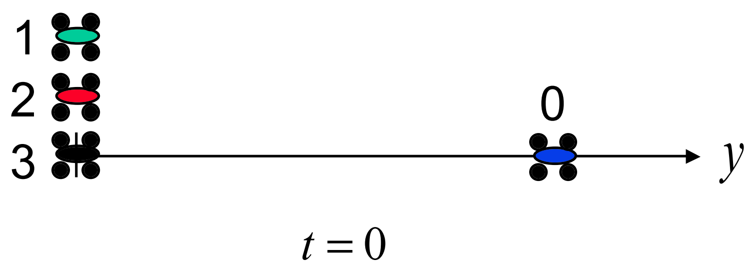
One-Dimensional Stability Problem

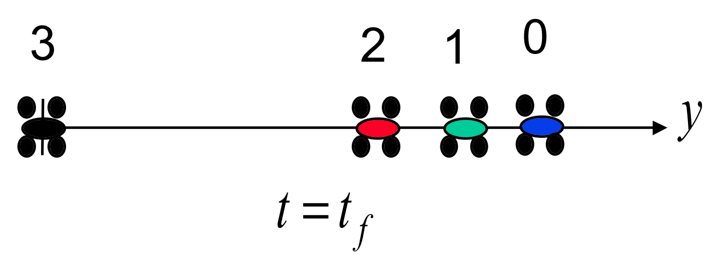
Problem 1: Spread out uniformly





Problem 2: Spread out in specified pattern



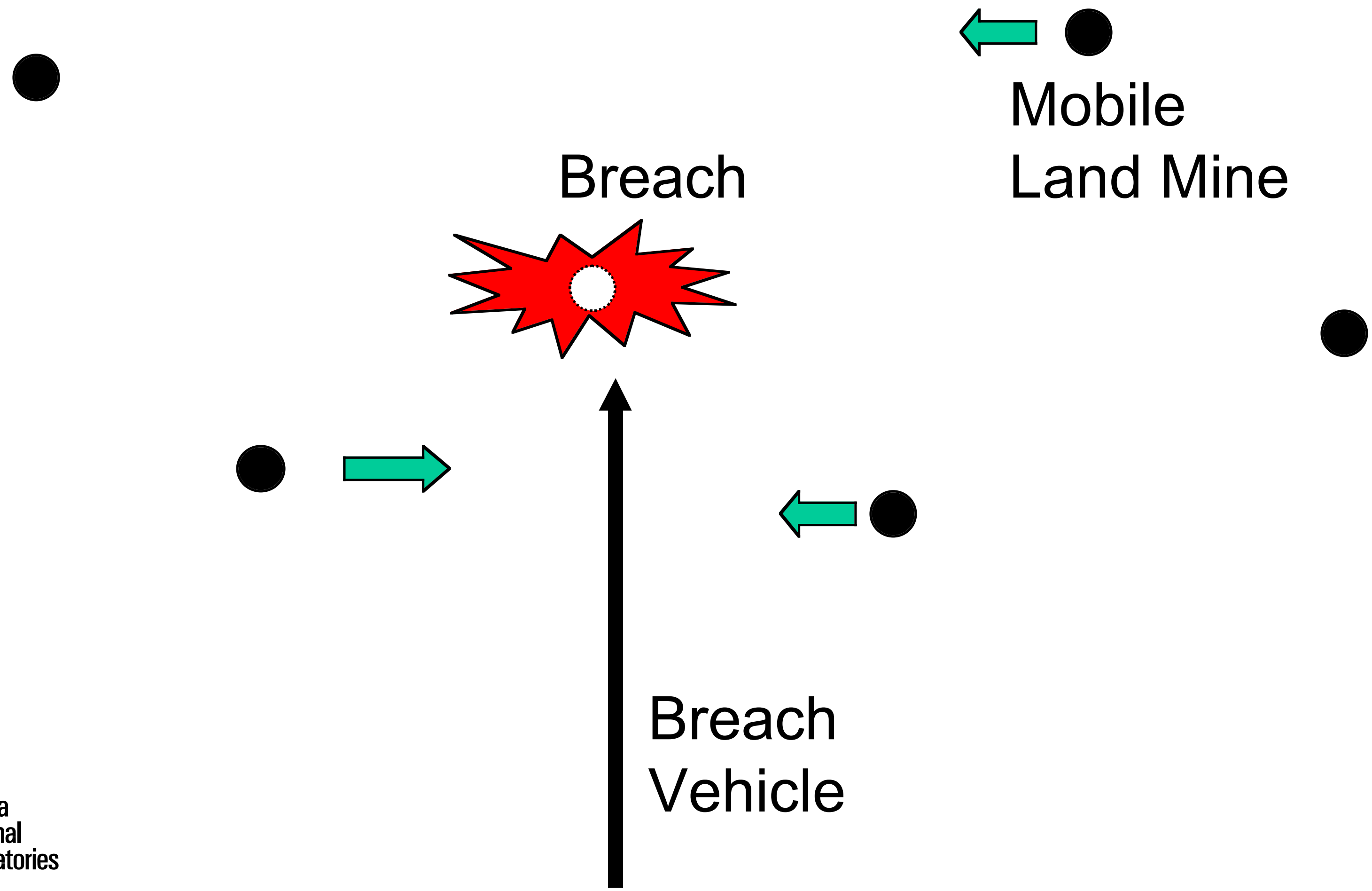


Two Dynamic Models:

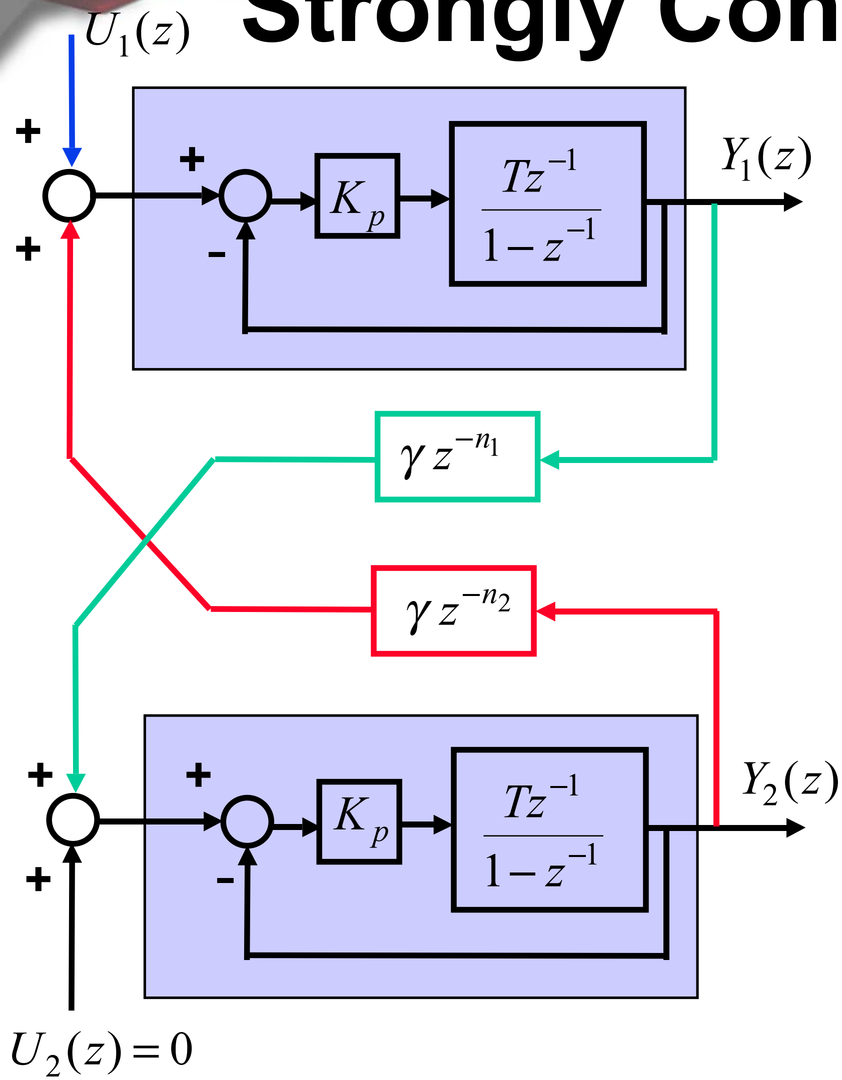
1. Localization sample period = Communication sample period
2. Localization sample period << Communication sample period

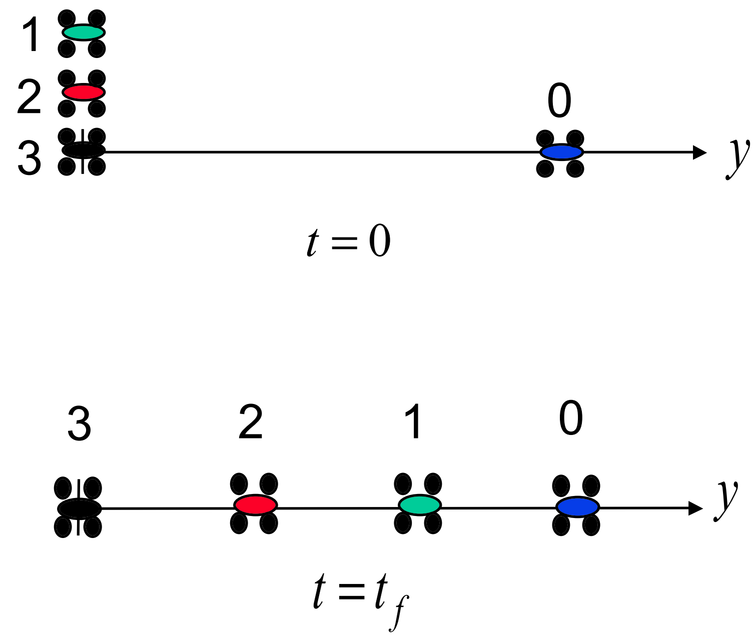
Perimeter Adjustment is 1D Problem

Self-Healing Minefield is 1D Problem



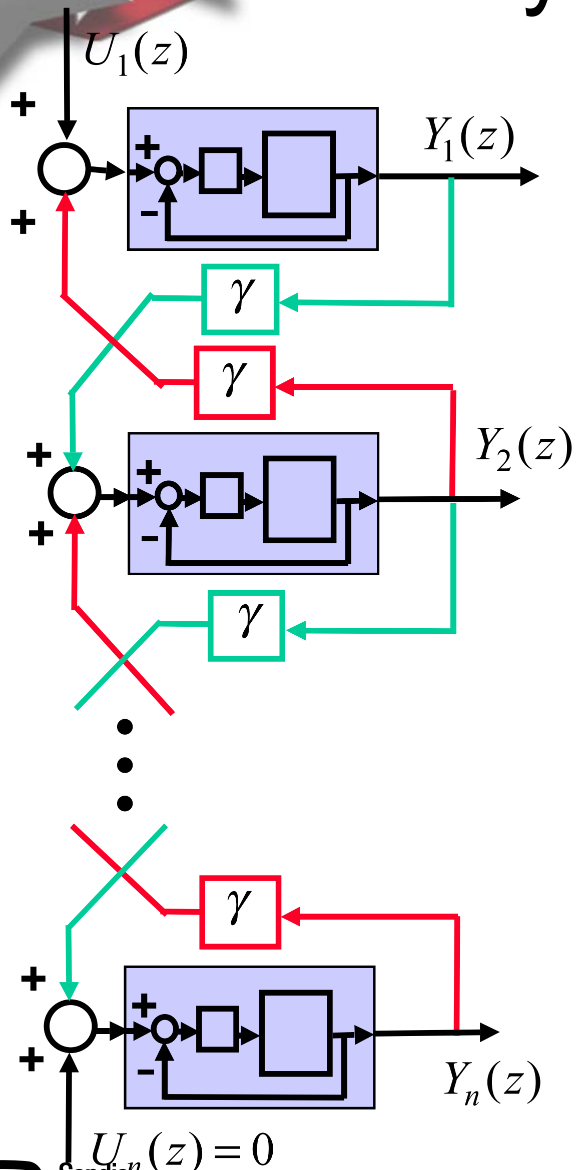
Model 1 for Two Strongly Connected Vehicles





T is the sample period of the position feedback loop. Assumes that position updates are the same as communication updates.

Stability using Vector Liapunov



Difference Equations

$$y_i(k+1) = A_i y_i(k) + A_{i(i-1)} y_{i-1}(k) + A_{i(i+1)} y_{i+1}(k)$$

for $i = 2, \ldots, n-1$

System is connectively globally asymptotically stable if matrix below is an M-matrix (leading principle minors are positive).

$$W = \begin{bmatrix} \xi_1 & -\varepsilon\xi_{12} & 0 & 0 \\ -\varepsilon\xi_{21} & \xi_2 & -\varepsilon\xi_{23} & 0 \\ 0 & -\varepsilon\xi_{32} & \xi_3 & O \\ 0 & 0 & O & O \end{bmatrix}$$

where

$$\xi_i = \frac{1}{\lambda_M(H_i^*) + \lambda_M^{1/2}(H_i^*)\lambda_M^{1/2}(H_i^* - I)}$$

$$\xi_{ij} = \lambda_M^{1/2}(A_{ij}^T A_{ij})$$

$$A_i^T H_i^* A_i - H_i^* = -I \qquad 0 \le \varepsilon \le 1$$

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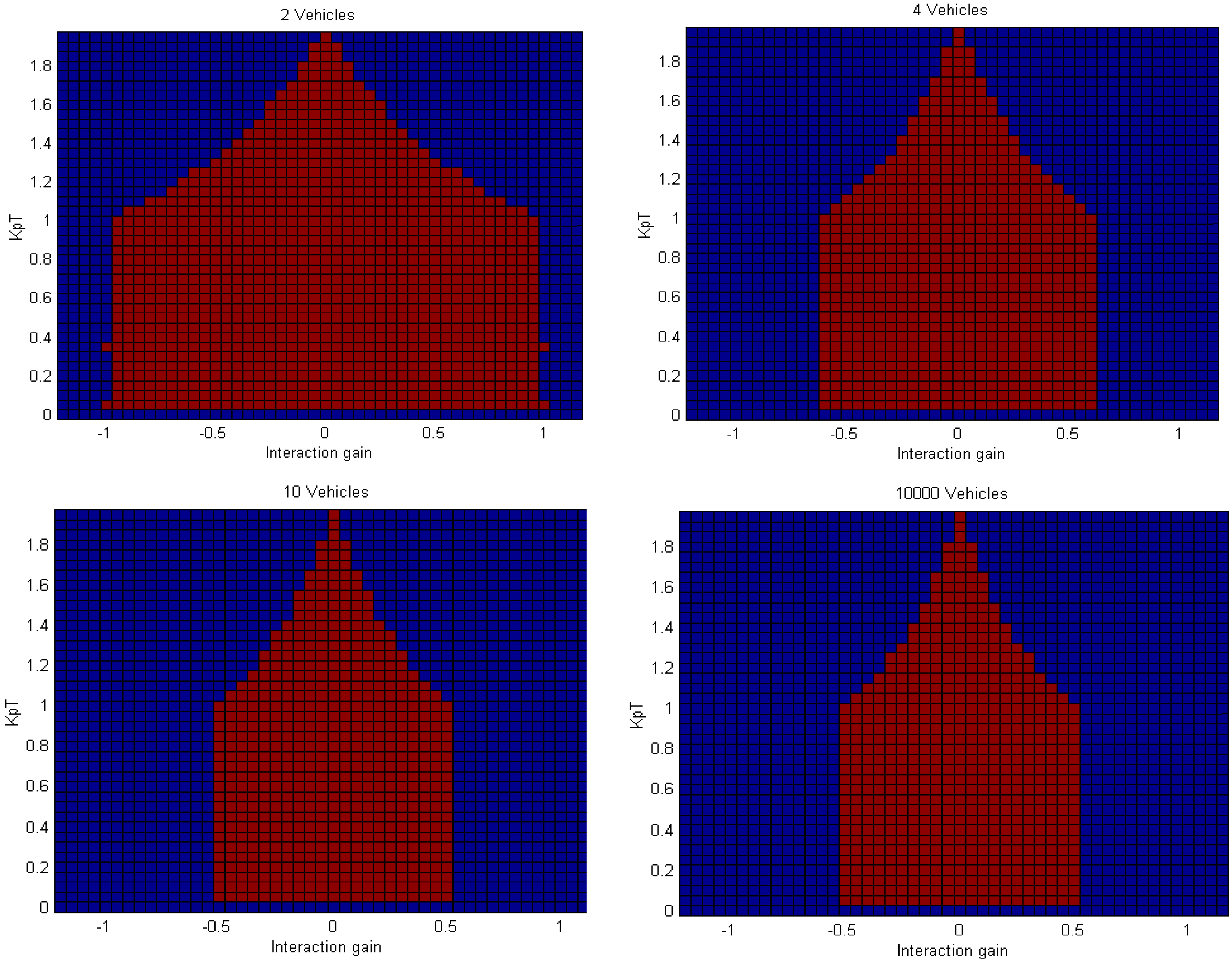
Model 1 Stable Regions for Multiple Vehicles



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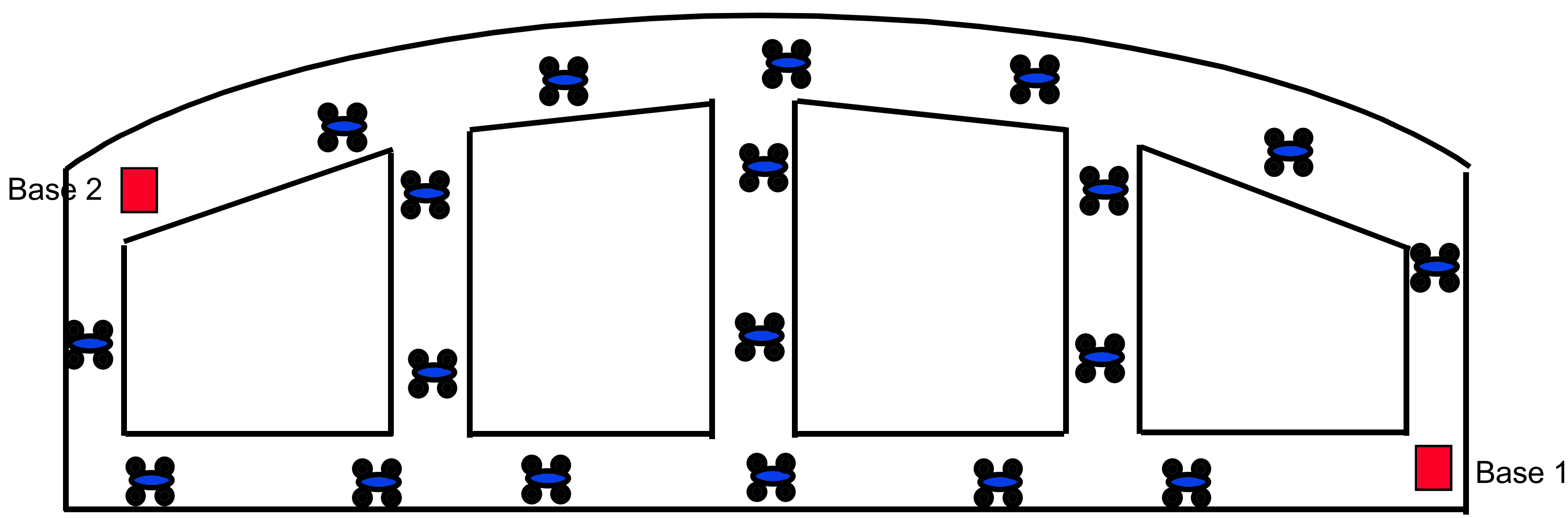
Model 1 Conclusion

- Stability depends on vehicle responsiveness K_p, communication sampling period T, and interaction gain γ.
- System goes unstable if communication sampling period is too long and/or vehicle responsiveness is too fast.
- Interaction gains can be used to contract or spread out vehicles.
- The stability region reaches a limit for large numbers of vehicles.

Two-Dimensional Problem: Communication/Navigation Network

Going into an unknown environment, and spreading out with uniform density.
Want to maintain communication between adjacent vehicles.



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Initial State


Goal: Communication/Navigation Network

2D Localization Algorithms Investigated

- Law of Cosines
- Steepest Descent
- Conjugate Gradient
- Least Squares
- Kaczmarz Distributed Algorithm

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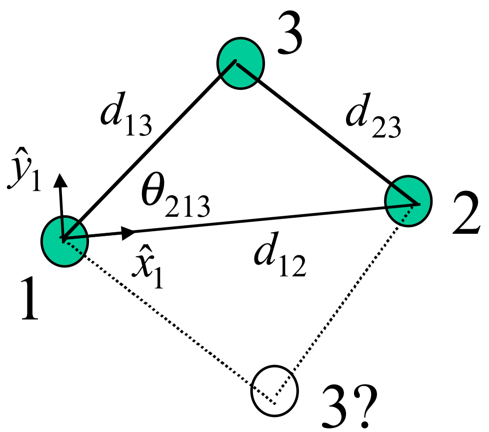
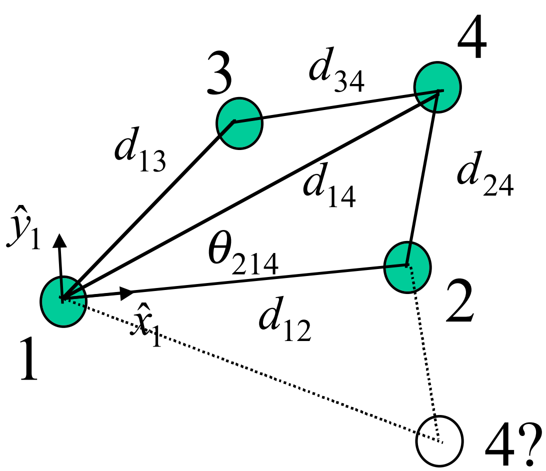
Law of Cosines



$$d_{23}^2 = d_{12}^2 + d_{13}^2 - 2d_{12}d_{13}\cos\theta_{213}$$

Can show that

$$^1x_3 = \frac{d_{12}^2 + d_{13}^2 - d_{23}^2}{2d_{12}} \qquad ^1y_3 = \pm\sqrt{d_{13}^2 - \left(^1x_3\right)^2}$$



Similarly

$$^1x_4 = \frac{d_{12}^2 + d_{14}^2 - d_{24}^2}{2d_{12}} \qquad ^1y_4 = \pm\sqrt{d_{14}^2 - \left(^1x_4\right)^2}$$

Select sign which minimizes

$$\left| d_{34}^2 - \left[\left(^1x_4 - {}^1x_3\right)^2 + \left(^1y_4 - {}^1y_3\right)^2 \right] \right|$$

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Steepest Descent Method

$$\min_{\bar{x}} f(\bar{x}) \quad \text{where} \quad f(\bar{x}) = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left(d_{ij}^2 - (x_i - x_j)^2 - (y_i - y_j)^2 \right)^2$$

Iterative Solution:

$$\bar{x}(k+1) = \bar{x}(k) - \alpha \nabla f(\bar{x}(k))$$

Do not need to know all d_{ij} !!

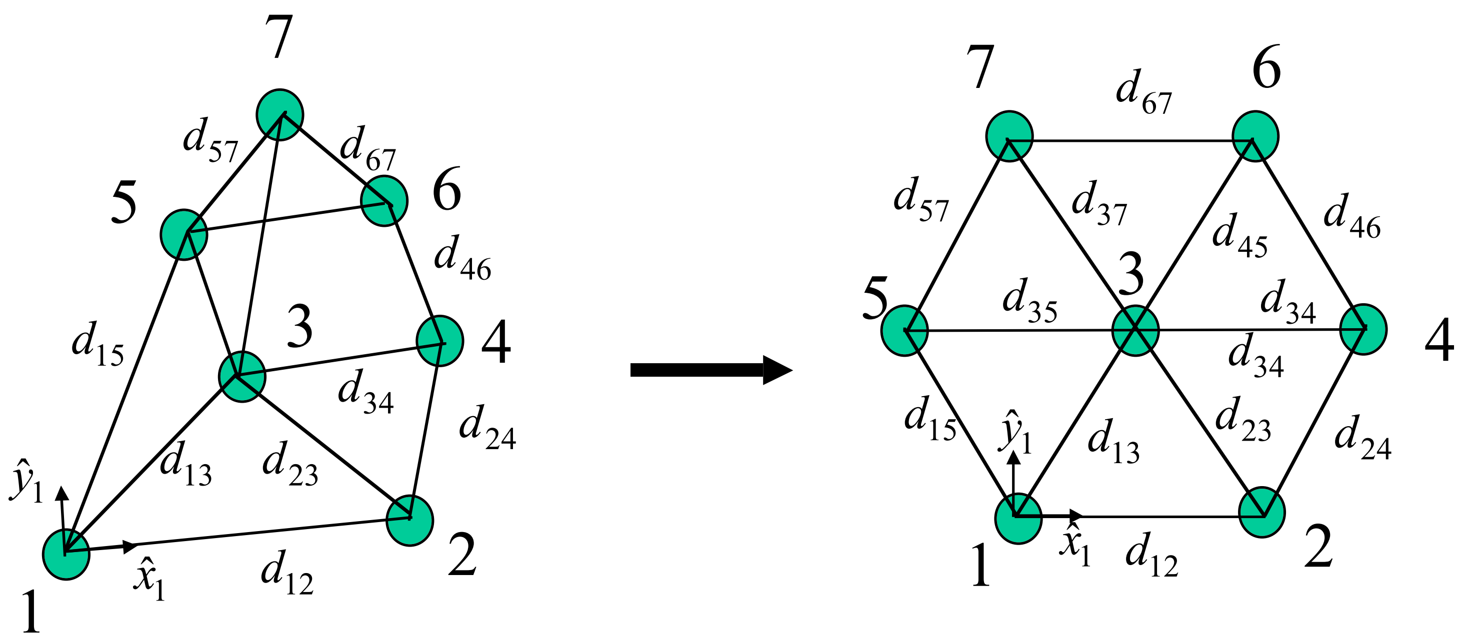
$$\bar{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \\ y_1 \\ \vdots \\ y_n \end{bmatrix} \in \Re^{2n} \qquad \nabla f = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \vdots \\ \frac{\partial f}{\partial x_n} \\ \frac{\partial f}{\partial y_1} \\ \vdots \\ \frac{\partial f}{\partial y_n} \end{bmatrix} \in \Re^{2n}$$

$$\frac{\partial f(\bar{x})}{\partial x_i} = -4 \sum_{\substack{j=1 \\ j \neq i}}^{n} \left[d_{ij}^2 - (x_i - x_j)^2 - (y_i - y_j)^2 \right] (x_i - x_j)$$

$$\frac{\partial f(\bar{x})}{\partial y_i} = -4 \sum_{\substack{j=1 \\ j \neq i}}^{n} \left[d_{ij}^2 - (x_i - x_j)^2 - (y_i - y_j)^2 \right] (y_i - y_j)$$

Minefield Algorithm

Swarming Behaviors Described as an Optimization Problem



Centralized: $\min_{\bar{x}} f(\bar{x})$ where $f(\bar{x}) = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left(d_{ij}^2 - (x_i - x_j)^2 - (y_i - y_j)^2 \right)^2$

or

Distributed: $\min_{\bar{x}} f_i(\bar{x}) \ \forall i$ where $f_i(\bar{x}) = \sum_{j \in NN} \left(d_{ij}^2 - (x_i - x_j)^2 - (y_i - y_j)^2 \right)^2$

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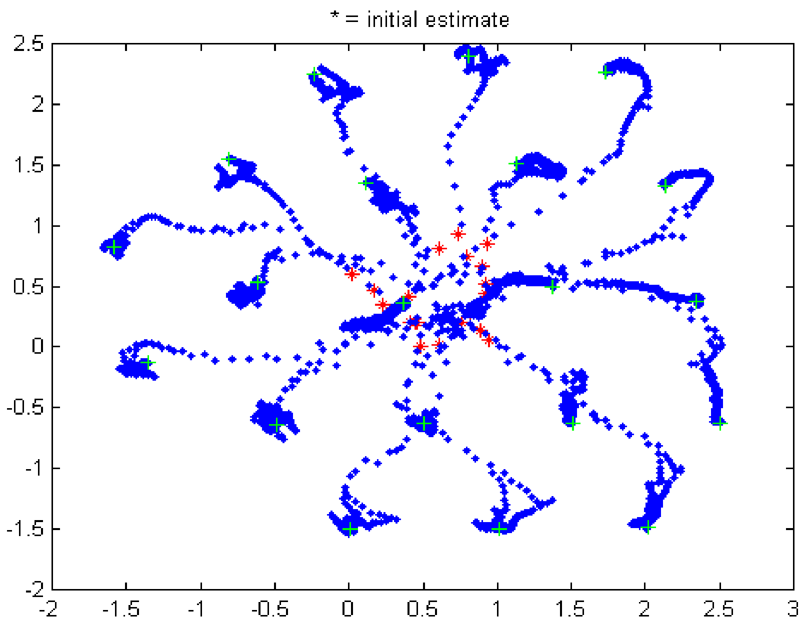
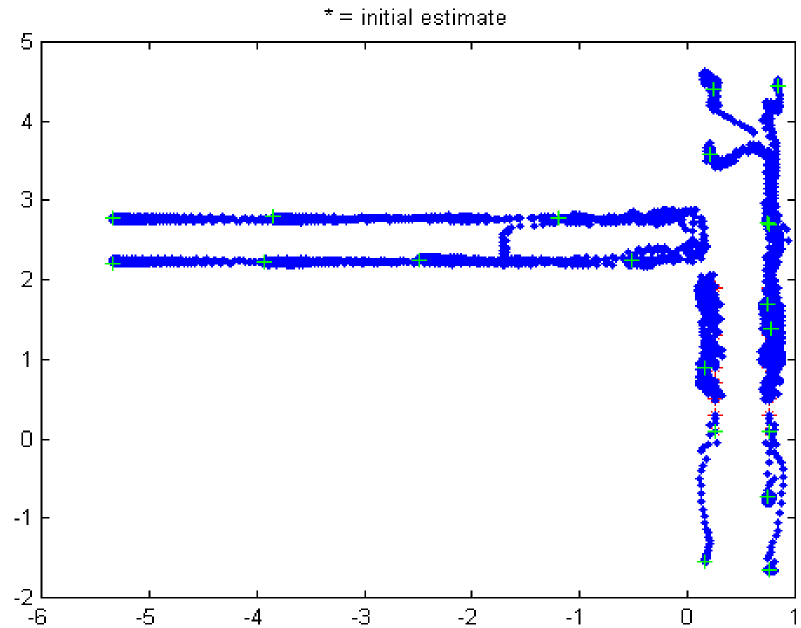
Dispersion Simulations



Dispersion using position of 3 nearest neighbors.



Dispersion in hallway.

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Swarming Behaviors Described as an Optimization Problem

Iterative Solution: $\bar{x}_i(k+1) = \bar{x}_i(k) - \alpha \nabla f_i(\bar{x}(k))$

$$\bar{x}_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix} \in \Re^2 \quad \nabla f = \begin{bmatrix} \dfrac{\partial f}{\partial x_i} \\ \dfrac{\partial f}{\partial y_i} \end{bmatrix} \in \Re^2$$

$$\frac{\partial f_i(\bar{x})}{\partial x_i} = -4 \sum_{j \in NN} \left[d_{ij}^2 - (x_i - x_j)^2 - (y_i - y_j)^2 \right] (x_i - x_j)$$

$$\frac{\partial f_i(\bar{x})}{\partial y_i} = -4 \sum_{j \in NN}^{n} \left[d_{ij}^2 - (x_i - x_j)^2 - (y_i - y_j)^2 \right] (y_i - y_j)$$

Meta-Level Behaviors such as:

- Dispersion
- Following
- Clustering
- Orbiting

can be mathematically described using this optimization approach with different values of d_{ij}.

Most importantly, we can prove stability and convergence of these solutions!!!

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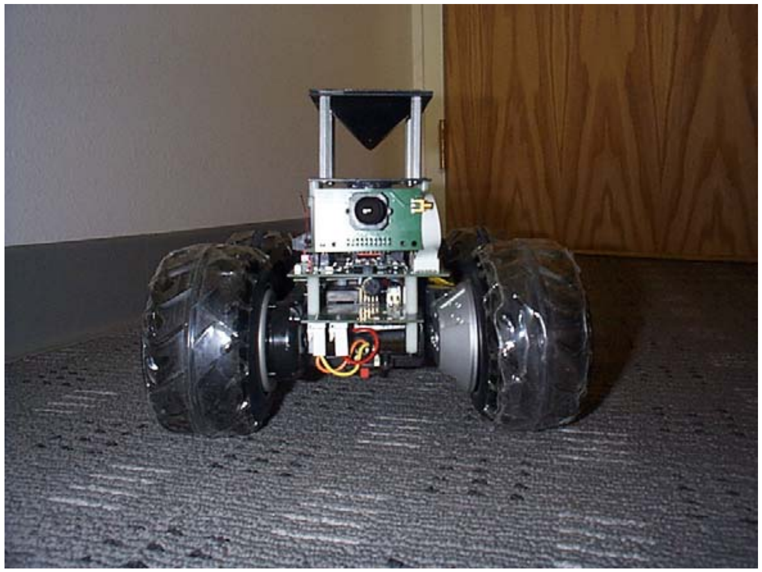
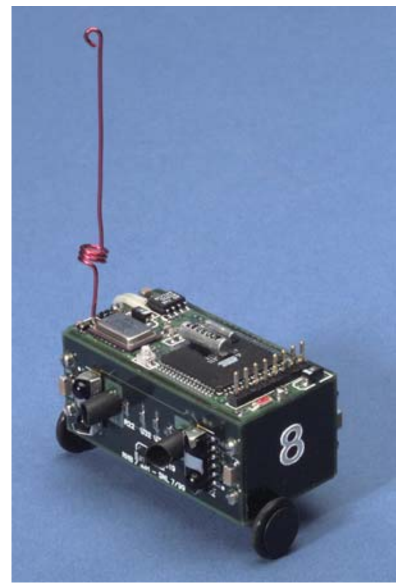
Gradient-Based Dispersion

Future Work

- Demonstrate communication/navigation network with 20 Netbot vehicles.
- Add surveillance cameras to Netbots.
- Extend to a heterogeneous indoor/outdoor communication/navigation network consisting of RATLERs, Netbots, and Millibots.
- Continue statistical and quantum mechanics analysis for 1000s.

Heterogeneous Team





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